Estimation of k-Factor Gigarch Process: A Monte Carlo Study

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ABSTRACT

In this paper, we discuss the parameter estimation for a k-factor generalized long memory process with conditionally heteroskedastic noise. Two estimation methods are proposed. The first method is based on the conditional distribution of the process and the second is obtained as an extension of Whittle's estimation approach. For comparison purposes, Monte Carlo simulations are used to evaluate the finite sample performance of these estimation techniques, **using four different conditional distribution functions**.

1. INTRODUCTION

Long-range dependence, as described by Mandelbrot and Van Ness (1968), or by Granger (1980), is present in many time series. One can think of time series in the domain of hydrology, climatology, medicine, astronomy or finance. To solve the parameter estimation problem of the generalized long memory process, several estimation procedures have been suggested in the literature. For example, Gray *et al.* (1989), Chung (1996, 1994) or Yajima (1996) and others proposed a two-step estimation procedure to estimate parameters of a generalized long memory process. In the first step, the estimation of the location of the singularities is dealt with, by using a grid-search procedure, or by taking the maximum of the periodogram. In the second step, the memory parameter is estimated by using classical parametric or semi-parametric methods of the long memory domain. Recently a simultaneous pseudo-maximum likelihood Whittle approximation has been proposed in order to estimate the parameters of the k-factor GARMA(p, d, ν, q) process, Ferrara and Guégan (2001), Ferrara (2000) or Giraitis and Leipus (1995). Moreover, Ferrara and Guégan studied a Monte Carlo simulation comparison for proposed parameter estimation methods. We note that all of the aforementioned works assume that the conditional variance of the time series is constant over time.

In the case of FARIMA(p, d, q) model with Gaussian distributed innovations, Reisen *et al.* (2001) have compared many parameter estimation methods. They indicated that the regression methods outperform the parametric Whittle's method when short memory parameters are involved. When the conditional variance follows an ARCH(r) model, the parameter estimation has been studied by Ling and Li (1997). They have developped the conditional sum of squares method of parameter estimation and have given the asymptotic properties of the estimated parameter. Baillie *et al.* (1996) applied this model to analyze the monthly inflation prices of different countries. Guégan (2000, 2003) introduced a new time varying volatility model, called the k-factor GIGARCH process. The parameter estimation of this process was carried out by Diongue and Guégan (2004). They proposed two pseudo-maximum likelihood parameter estimation methods and for each of these methods they investigated the asymptotic properties of the estimators. Finally, an application on electricity market spot prices was proposed by Diongue *et al.* (2004) or Diongue (2005).

The main objective of this paper is to evaluate the performance via Monte Carlo simulations for the two proposed parametric estimation methodologies for the k-factor GIGARCH(p, d, ν, q) model introduced in Diongue and Guégan (2004). For instance, we consider here the conditional sum of squares approach when the distribution of the disturbances is normal, Student-t, Ling and Li (1997) and Diongue and Guégan (2004), **GED, Harvey (1981) and Box and Tiao (1973), and Skew Student-t, Hansen (1994) and Fernandez and Steel (1998). Indeed, it is widely accepted that financial returns, on a weekly, daily or intraday basis, are fat-tailed and even skewed, Peiró (1999)**. For comparison purpose, as Ferrara and Guégan (2001) suggested that the estimators obtained by maximum likelihood method converge more quickly than those given by semi-parametric procedure, the parametric Whittle maximum likelihood estimator is included in the simulation study.

The article is organized as follows. In section 2, we present the k-factor GIGARCH model and give some important assumptions. Section 3 adresses Whittle parameter estimation as well as CSS procedure. Section 4 reports the results of several simulation experiments studying the behavior of the estimation procedures **for the four models**. Section 5 concludes.

2. THE K-FACTOR GIGARCH(p, d, ν, q) MODEL

In this section, we introduce the model, we will work with. Assume that $(\xi_t)_{t\in\mathbb{Z}}$ is a white noise process with unit variance and mean zero. Let $\phi(B) = 1 - \sum_{j=1}^{p} \phi_j B^j$ and $\theta(B) = 1 - \sum_{j=1}^{q} \theta_j B^j$ denote the ARMA operators and have no common roots. Assume that all the roots of the polynomials $\phi(B)$ and $\theta(B)$ lie outside the unit circle. Let B denotes the back shift operator, k a nonnegative integer, and d_j and ν_j be such that $0 < d_j < \frac{1}{2}$ if $|\nu_j| < 1$ or $0 < d_j < \frac{1}{4}$ if $|\nu_j| = 1$ for all $j = 1, \cdots, k$. We define a centered k-factor GIGARCH process

$(X_t)_{t\in\mathbb{Z}}$ by,

$$\phi(B)\prod_{j=1}^{k}\left(I-2\nu_j B+B^2\right)^{d_j}X_t=\theta(B)\,\varepsilon_t, \tag{1}$$

where

$$\varepsilon_t=\sqrt{h_t}\xi_t \;\text{ with }\; h_t=a_0+\sum_{j=1}^{r}a_j\varepsilon_{t-j}^2+\sum_{j=1}^{s}b_jh_{t-j}\;\text{ for all t,} \tag{2}$$

with $a_0>0$, $a_1,\cdots,a_r,b_1,\cdots,b_s\geq 0$ and $\sum_{j=1}^{r}a_j+\sum_{j=1}^{s}b_j<1$ where r and s are nonnegative integers. The frequencies $\lambda_j=\arccos(\nu_j)$ for all $j=1,\cdots,k$ are called the Gegenbauer frequencies (or G-frequencies). The process defined in (1)-(2) was introduced by Guégan (2000, 2003), generalizing in that way the fractionally integrated process with generalized autoregressive conditional heteroskedasticity disturbances (ARFIMA(p,d,q)-GARCH(r,s)) proposed by Baillie *et al.* (1996) and Ling and Li (1997). Note that the parameters which appear in (2) are short memory parameters but they distinguish the variance behavior of the process. It is therefore important to note that the model defined in (1)-(2) contains long memory and short memory parameters in the same time.

In this paper, we assume that $(X_t)_{t\in\mathbb{Z}}$ is a linear process without a deterministic term. We now define $U_t=\prod_{j=1}^{k}\left(I-2\nu_jB+B^2\right)^{d_j}X_t$, so that the process $(U_t)_{t\in\mathbb{Z}}$ is an ARMA(p,q)-GARCH(r,s) process, Ling and Li (1997) and Weiss (1986).

We recall that the Gegenbauer polynomials, often used in applied mathematics because of their orthogonality and recursion properties, are defined by :

$$\left(1-2\nu z+z^2\right)^{-d}=\sum_{j\geq 0}C_j(d,\nu)\,z^j, \tag{3}$$

where $|z|\leq 1$ and $|\nu|\leq 1$.

The coefficients $(C_j(d,\nu))_{j\in\mathbb{Z}}$ of this development can be computed in many different ways. For example, Rainville (1960) shows that :

$$C_j(d,\nu)=\sum_{k=0}^{\left[\frac{j}{2}\right]}\frac{(-1)^k\,\Gamma(d+j-k)\,(2\nu)^{j-2k}}{\Gamma(d)\,\Gamma(k+1)\,\Gamma(j-2k+1)}, \tag{4}$$

where $\left[\frac{j}{2}\right]$ is the integer part of $\frac{j}{2}$ and Γ the Euler gamma function defined by $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$. A more easy way to compute the Gegenbauer polynomials $(C_j(d,\nu))_{j\in\mathbb{Z}}$ is based on the following recursion formula :

$$\begin{aligned}C_j(d,\nu) &= 2\nu\left(\frac{d-1}{j}+1\right)C_{j-1}(d,\nu)\\&\quad-\left(2\frac{d-1}{j}+1\right)C_{j-2}(d,\nu),\forall\, j>1,\end{aligned} \tag{5}$$

with $C_0(d,\nu)=1$ and $C_1(d,\nu)=2d\nu$.

The process defined in (1)-(2) is stationary and invertible, Guégan (2000) and (2003), and its spectral density function, $f_X(\omega)$, is given by

$$f_X(\omega)=\prod_{j=1}^{k}\left|2\left[\cos(\omega)-\nu_j\right]\right|^{-2d_j} f_U(\omega), \tag{6}$$

where $f_U(\omega)$ is the spectral density function of the process $(U_t)_{t\in\mathbb{Z}}$ and $-\pi\le\omega\le\pi$.

3. ESTIMATION METHOD

In this section, we consider two methods for estimating the parameters of a k-factor GIGARCH(p,d,ν,q) process. The first one is based on the conditional sum of squares procedure while the second method deals with a parametric method proposed by Whittle.

3.1 CONDITIONAL SUM OF SQUARES ESTIMATION

Given a stationary k-factor GIGARCH process $\{X_t\}_{t=1}^{T}$ defined by equations (1)-(2). We denote by $\gamma=(\phi_1,\cdots,\phi_p,\theta_1,\cdots,\theta_q,d_1,\cdots,d_k)$, $\delta=(a_0,a_1,\cdots,a_r,b_1,\cdots,b_s)$ and $\omega=(\gamma,\delta)$ its parameters. We assume that $\omega_0=(\gamma_0,\delta_0)$ is the true value of ω and that ω_0 is in the interior of the compact set $\Theta\subseteq\mathbb{R}^{p+q+k+r+s+1}$. The conditional sum of squares estimator $\hat{\omega}_T$ of ω in Θ maximizes the conditional logarithmic likelihood $L(\omega)$ on F_0, where F_t is the σ-algebra generated by $(X_s, s\le t)$. **We give now the expressions of $L(\omega)$ for the four models that we are using in the simulation experiments**.

1. If we assume that the innovations $(\varepsilon_t)_{t\in\mathbb{Z}}$ have a conditional Gaussian distribution then the conditional log-likelihood is defined by :

$$L(\omega)=-\frac{T}{2}\log(2\pi)-\frac{1}{2}\sum_{t=1}^{T}\left[\log(h_t)+\frac{\varepsilon_t^2}{h_t}\right]. \tag{7}$$

2. Now, if we assume that the innovations $(\varepsilon_t)_{t\in\mathbb{Z}}$ have a conditional Student-t distribution with l degrees of freedom, then the CSS estimator $\hat{\omega}_T$ maximizes the log-likelihood function $L(\omega)$ defined by

$$\begin{aligned} L(\omega) = T &\left[\log\Gamma\left\{\frac{(l+1)}{2}\right\} - \log\Gamma\left(\frac{l}{2}\right) - \frac{1}{2}\log(l-2)\right] \\ &- \frac{1}{2}\sum_{t=1}^{T}\left\{\log(h_t) + (l+1)\log\left[1+\frac{\varepsilon_t^2}{h_t(l-2)}\right]\right\}. \end{aligned} \tag{8}$$

3. **In 1991, Nelson suggested to consider the family of GED distribution. The probability density function, $f(.)$, of a normalized GED random variable is given by :**

$$f(x) = \frac{l2^{-\left(1+\frac{1}{l}\right)}}{\lambda_l\Gamma\left(\frac{1}{l}\right)}e^{-\frac{1}{2}\left|\frac{x}{\lambda_l}\right|^l}, \quad -\infty < x < \infty, \tag{9}$$

with $\lambda_l = \sqrt{\frac{\Gamma\left(\frac{1}{l}\right)2^{-\frac{2}{l}}}{\Gamma\left(\frac{3}{l}\right)}}$ **and** $0 < l < \infty$ **is the tail-thickness parameter. The GED includes the Gaussian distribution ($l = 2$) as a special case, along with many other distributions, some more fat-tailed than the Gaussian one (e.g. the double exponential distribution corresponding to $l = 1$) and some more thin-tailed (e.g the Uniform distribution on the interval $\left[-\sqrt{3}, \sqrt{3}\right]$ when $l \to \infty$). The GED log-likelihood function of a normalized random variable is given by :**

$$\begin{aligned} L(\omega) = T &\left[\log\left(\frac{l}{\lambda_l}\right) - \left(1+\frac{1}{l}\right)\log(2) - \log\Gamma\left(\frac{1}{l}\right)\right] \\ &-\frac{1}{2}\sum_{t=1}^{T}\left[\log(h_t) + h_t^{-\frac{l}{2}}\left|\frac{\varepsilon_t}{\lambda_l}\right|^l\right]. \end{aligned} \tag{10}$$

4. **Hansen (1994) pointed out that the conditional distribution of innovations may not be only leptokurtic but also asymmetric, and then proposed the skewed Student's t density function defined as follows :**

$$f(x) = \begin{cases} bc\left[1+\frac{1}{l-2}\left(\frac{bx+a}{1-\zeta}\right)^2\right] & \textbf{if } x < -\frac{a}{b} \\ \\ bc\left[1+\frac{1}{l-2}\left(\frac{bx+a}{1+\zeta}\right)^2\right] & \textbf{if } x \geq -\frac{a}{b}, \end{cases} \tag{11}$$

where $2 < l < \infty$ **and** $-1 < \zeta < 1$**. The constants** a**,** b **and** c **are given by**

$$a = 4\zeta c \frac{l-2}{l-1}, \; b^2 = 1 + 3\zeta^2 - a^2, \; \textbf{and } c = \frac{\Gamma\left(\frac{l+1}{2}\right)}{\sqrt{\pi\,(l-2)}\Gamma\left(\frac{l}{2}\right)}.$$

By setting $\zeta = 0$**, this density function simply turns out to be the Student-t distribution. A very similar version of this skewed Student's t distribution was introduced independently by Fernandez and Steel (1998). Other alternatives of the skew Student-t distribution have been proposed in the literature, Jones and Faddy (2003) and Azzalini and Capitanio (2003). The log-likelihood is given by**

$$L(\omega) = T\log c + T\log b - \frac{1}{2}\sum_{t=1}^{T}\left\{\log(h_t) + (1+l)\log\left[1 + \frac{1}{(l-2)}\frac{\left(b\frac{\varepsilon_t}{\sqrt{h_t}} + a\right)^2}{(1+\zeta I_t)}\right]\right\}, \tag{12}$$

where $I_t = \begin{cases} -1 & \textbf{if } \frac{\varepsilon_t}{\sqrt{h_t}} < -\frac{a}{b} \\ 1 & \textbf{if } \frac{\varepsilon_t}{\sqrt{h_t}} \geq -\frac{a}{b}. \end{cases}$

The asymptotic properties of the estimators were given in Diongue and Guégan (2004) when the disturbances are symmetrically distributed with finite fourth moment (Normal and Student-t cases). **However, our result could be very easily extended to the GED case. Moreover, in the skew Student-t case the distribution may not be symmetric (**$\zeta \neq 0$**), thus it is necessary to study the asymptotic properties theory of the CSS estimator. This will be done in a compagnon paper.**

3.2 WHITTLE ESTIMATION

In this paragraph, we investigate the sequential Whittle's method to estimate all parameters of the process $\{X_t\}_{t=1}^{T}$ defined by equations (1)-(2).

1. The first step consists of estimation of the long-memory parameters $d = (d_1, \cdots, d_k)$ and the ARMA(p, q) parameters $\alpha = (\phi_1, \cdots, \phi_p, \theta_1, \cdots, \theta_q)$ using Whittle's approach, Chung (1994, 1996) and Ferrara and Guégan (2001). Let be $\hat{\gamma} = \left(\hat{d}_1, \cdots, \hat{d}_k, \hat{\phi}_1, \cdots, \hat{\phi}_p, \hat{\theta}_1, \cdots, \hat{\theta}_q\right)$

the Whittle estimator. It is based on the periodogram and it involves the function

$$L_T(\gamma) = \frac{1}{2T}\sum_{j=1}^{T-1}\left\{\log\left[f_X(\omega_j,\gamma)\right] + \frac{I_X(\omega_j)}{f_X(\omega_j,\gamma)}\right\}, \tag{13}$$

where $I_X(\omega_j)$ is the periodogram of the process $(X_t)_{t\in\mathbb{Z}}$ and expresses as follows

$$I_X(\omega_j) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} X_t^2 e^{i\omega_j t}\right|^2, \tag{14}$$

with the frequencies ω_j are such that $\omega_j = \frac{2\pi j}{T}$, $0 \le j \le \left[\frac{T}{2}\right]$, $\left[\frac{T}{2}\right]$ the integer part of $\frac{T}{2}$ and i a complex number such that $i^2 = -1$. The function $f_X(\omega_j,\gamma)$ is defined by

$$f_X(\omega_j,\gamma) = \left|\frac{\theta(e^{-i\omega_j})}{\phi(e^{-i\omega_j})}\right|^2 \prod_{l=1}^{k}\left|2\left[\cos(\omega_j) - \nu_l\right]\right|^{-2d_l}. \tag{15}$$

Diongue and Guégan (2004) have shown that the maximum likelihood of γ is consistent and asymptotically normally distributed.

2. In the second step, the GARCH(r,s) parameters $\delta = (a_0, a_1, \cdots, a_r, b_1, \cdots, b_s)$ are estimated using Whittle's method applied to the residuals of the long-memory process, Giraitis and Robinson (2001). **We follow the idea developed by Bollerslev (1986) which has pointed out that the process $(\varepsilon_t^2)_{t\in\mathbb{Z}}$ generated by (2) has an ARMA**$(\max(r,s), s)$ **representation expressed as follows :**

$$\varphi(B)\,\varepsilon_t^2 = \psi(B)\,\upsilon_t, \tag{16}$$

where the polynomials $\varphi(B)$ and $\psi(B)$ are defined by $\varphi(B) = 1 - \sum_{j=1}^{\max(r,s)}(a_j + b_j)B^j$ and $\psi(B) = 1 - \sum_{j=1}^{s} b_j B^j$, respectively. Notice that in $\varphi(B)$, we set $b_j = 0$ if $j \in (s,r]$ and $a_j = 0$ when $j \in (r,s]$. $(\upsilon_t)_{t\in\mathbb{Z}}$ are martingale differences defined by $\upsilon_t = \varepsilon_t^2 - h_t$ for all t.
Thus, the estimator $\hat{\delta} = \left(\hat{a_0}, \hat{a_1}, \cdots, \hat{a_r}, \hat{b_1}, \cdots, \hat{b_s}\right)$ is obtained by maximizing the function $L_T(\delta)$ that is given by

$$L_T(\delta) = \frac{1}{2T}\sum_{j=1}^{T-1}\left\{\log\left[f(\omega_j,\delta)\right] + \frac{I_\varepsilon(\omega_j)}{f(\omega_j,\delta)}\right\}. \tag{17}$$

where $I_{\varepsilon}(\omega_j)$ is the periodogram of the process $(\varepsilon_t^2)_{t\in\mathbb{Z}}$ and $f(\omega_j,\delta)$ its spectral density defined by

$$f(\omega_j,\delta)=\frac{\sigma^2}{2\pi}\left|\frac{\psi\left(e^{-i\omega_j}\right)}{\varphi\left(e^{-i\omega_j}\right)}\right|^2, \tag{18}$$

and $\sigma^2=E\left(\upsilon_t^2\right)$.

4. MONTE CARLO SIMULATION

In order to access the finite sample performance of the methods described previously in the context of generalized long memory models with conditional heteroskedastic noises, some Monte Carlo experiments were carried out. The simulations are based on a generalized long memory process with only one explosion and time varying volatility following an ARCH(1) model with Normal, Student-t, **GED and skew Student-t** error distributions. Thus, the model is defined as follows

$$\left(I-2\nu B+B^2\right)^d X_t=\varepsilon_t, \tag{19}$$

where

$$\varepsilon_t=\sqrt{h_t}\xi_t,\quad\text{and}\quad h_t=a_0+a_1\varepsilon_{t-1}^2. \tag{20}$$

The models and parameter values are specified in the tables which also give the empirical mean, mean absolute error (MAE) and the roots mean squared error (RMSE) of the estimation procedures based on 500 replications of series with sample sizes $T=500$ and 1000. Throughout all simulation experiments, we set $\nu=\cos\left(\frac{\pi}{6}\right)$. All calculations were carried out using Matlab version 6.1 Toolbox. The k-factor GIGARCH(p,d,ν,q) processes were simulated following the numerical method developed in Beran (1994), Ferrara (2000), or Diongue (2005).

Table I

Estimator parameters for the centered Gaussian GIGARCH model (500 replications) defined by (19)-(20).

True value			CSS method			Whittle's method		
d	a_0	a_1	$\hat{d}$	$\hat{a}_0$	$\hat{a}_1$	$\hat{d}$	$\hat{a}_0$	$\hat{a}_1$
			$T = 500$					
0.25	0.6	0.4	0.2484	0.6068	0.3909	0.2670	0.6101	0.3938
			(0.0204)	(0.0455)	(0.0646)	(0.0297)	(0.0489)	(0.0668)
			[0.0257]	[0.0577]	[0.0805]	[0.0377]	[0.0620]	[0.0855]
0.3	0.2	0.3	0.2996	0.2009	0.2944	0.356	0.200	0.295
			(0.0193)	(0.0142)	(0.0576)	(0.0572)	(0.0134)	(0.0602)
			[0.0247]	[0.0178]	[0.0725]	[0.0635]	[0.0165]	[0.0785]
0.35	0.8	0.5	0.3484	0.8014	0.500	0.3633	0.8091	0.4894
			(0.01756)	(0.06509)	(0.0759)	(0.0328)	(0.0635)	(0.0709)
			[0.0229]	[0.0816]	[0.0961]	[0.0405]	[0.0806]	[0.0901]
			$T = 1000$					
0.25	0.6	0.4	0.2507	0.6027	0.3929	0.2580	0.6039	0.3958
			(0.0129)	(0.0322)	(0.0451)	(0.0189)	(0.0342)	(0.0469)
			[0.0162]	[0.0408]	[0.0570]	[0.0239]	[0.0424]	[0.0576]
0.3	0.2	0.3	0.2995	0.2006	0.2998	0.3272	0.2015	0.2942
			(0.01427)	(0.0105)	(0.0415)	(0.0295)	(0.0105)	(0.0432)
			[0.0175]	[0.0132]	[0.0519]	[0.0346]	[0.0136]	[0.0531]
0.35	0.8	0.5	0.3485	0.7992	0.4994	0.3565	0.8015	0.4954
			(0.0132)	(0.0443)	(0.0495)	(0.0212)	(0.0448)	(0.0487)
			[0.0167]	[0.0585]	[0.0618]	[0.0271]	[0.0567]	[0.0607]

Table II

Estimator parameters for the centered Student-t GIGARCH model with $l = 5$ degree of freedom (500 replications) defined by (19)-(20).

True value				CSS method				Whittle's method			
d	a_0	a_1	l	$\hat{d}$	$\hat{a}_0$	$\hat{a}_1$	$\hat{l}$	$\hat{d}$	$\hat{a}_0$	$\hat{a}_1$	$\hat{l}$
						$T = 500$					
0.25	0.6	0.4	5	0.2489	0.5998	0.4006	5.5405	0.2677	0.6140	0.3900	5.4354
				(0.0189)	(0.0652)	(0.0928)	(0.1117)	(0.036)	(0.066)	(0.0932)	(0.1061)
				[0.0238]	[0.0807]	[0.1156]	[0.1741]	[0.0465]	[0.0845]	[0.1185]	[0.1535]
0.3	0.2	0.3	5	0.3017	0.2018	0.2956	5.5806	0.3581	0.2021	0.2993	5.6278
				(0.0191)	(0.0212)	(0.0876)	(0.1163)	(0.0619)	(0.0222)	(0.0843)	(0.1133)
				[0.0242]	[0.0271]	[0.1077]	[0.1688]	[0.0705]	[0.0282]	[0.107]	[0.1704]
0.35	0.8	0.5	5	0.3486	0.8153	0.4942	5.5434	0.3610	0.8150	0.4943	5.5653
				(0.0167)	(0.0911)	(0.1053)	(0.1163)	(0.0384)	(0.0926)	(0.1029)	(0.1199)
				[0.0213]	[0.1161]	[0.1319]	[0.2099]	[0.0050]	[0.1181]	[0.1275]	[0.2063]
						$T = 1000$					
0.25	0.6	0.4	5	0.2487	0.6019	0.4019	5.2461	0.2607	0.6074	0.3933	5.2435
				(0.0141)	(0.0456)	(0.0647)	(0.6825)	(0.0272)	(0.0461)	(0.0631)	(0.7043)
				[0.0174]	[0.0584]	[0.0831]	[0.9106]	[0.036]	[0.0585]	[0.0793]	[0.9468]
0.3	0.2	0.3	5	0.2999	0.2016	0.3001	5.2137	0.3274	0.2016	0.2996	5.1929
				(0.0137)	(0.0157)	(0.0578)	(0.7320)	(0.0324)	(0.0151)	(0.0531)	(0.6793)
				[0.0171]	[0.0197]	[0.0723]	[0.1005]	[0.0403]	[0.0194]	[0.0679]	[0.1029]
0.35	0.8	0.5	5	0.3491	0.8073	0.4960	5.2491	0.3543	0.8035	0.4899	5.3206
				(0.0127)	(0.0634)	(0.0666)	(0.7023)	(0.0277)	(0.0653)	(0.0721)	(0.7485)
				[0.0158]	[0.0794]	[0.0848]	[0.1012]	[0.0378]	[0.0818]	[0.0910]	[0.1019]

Table III

Estimator parameters for the centered GED GIGARCH model with $l = 1.5$ degree of freedom (500 replications) defined by (19)-(20).

True value				CSS method				Whittle's method			
d	a_0	a_1	l	$\hat{d}$	$\hat{a}_0$	$\hat{a}_1$	$\hat{l}$	$\hat{d}$	$\hat{a}_0$	$\hat{a}_1$	$\hat{l}$
						$T = 500$					
0.25	0.6	0.4	1.5	0.2505	0.6049	0.3974	1.5306	0.2683	0.6073	0.3857	1.5289
				(0.0192)	(0.0499)	(0.0781)	(0.1181)	(0.0319)	(0.0505)	(0.0772)	(0.1174)
				[0.0236]	[0.0619]	[0.0974]	[0.1534]	[0.0406]	[0.0642]	[0.0957]	[0.1481]
0.3	0.2	0.3	1.5	0.2984	0.1997	0.2955	1.5260	0.3547	0.2035	0.2917	1.537
				(0.0201)	(0.0167)	(0.0697)	(0.1193)	(0.0582)	(0.0164)	(0.0734)	(0.1140)
				[0.0252]	[0.0211]	[0.087]	[0.1534]	[0.0654]	[0.0209]	[0.0920]	[0.1497]
0.35	0.8	0.5	5	0.3509	0.8015	0.4953	1.5329	0.364	0.810	0.4889	1.532
				(0.0176)	(0.0729)	(0.0811)	(0.1163)	(0.0334)	(0.0738)	(0.0922)	(0.1183)
				[0.0223]	[0.0914]	[0.1031]	[0.1489]	[0.0430]	[0.0947]	[0.1133]	[0.1505]
						$T = 1000$					
0.25	0.6	0.4	1.5	0.2496	0.6024	0.3964	1.5115	0.2571	0.6002	0.3934	1.5154
				(0.0138)	(0.0362)	(0.0517)	(0.0787)	(0.0212)	(0.0358)	(0.0520)	(0.0769)
				[0.0173]	[0.0468]	[0.0647]	[0.1003]	[0.0276]	[0.044]	[0.0668]	[0.0969]
0.3	0.2	0.3	1.5	0.2989	0.2005	0.2924	1.5121	0.3254	0.2016	0.2958	1.5150
				(0.0141)	(0.0108)	(0.0506)	(0.0800)	(0.0284)	(0.0114)	(0.0458)	(0.0803)
				[0.0175]	[0.0138]	[0.0629]	[0.1023]	[0.0339]	[0.0144]	[0.0570]	[0.1048]
0.35	0.8	0.5	1.5	0.3505	0.8012	0.4956	1.5165	0.3577	0.7993	0.5007	1.5095
				(0.0118)	(0.0468)	(0.0631)	(0.0845)	(0.0246)	(0.0505)	(0.0591)	(0.0799)
				[0.0148]	[0.0582]	[0.0789]	[0.1064]	[0.0317]	[0.0626]	[0.0742]	[0.1012]

Table IV

Estimator parameters for the centered skew Student-t GIGARCH model with $l = 3$ degree of freedom and $\zeta = 0.5$ (500 replications) defined by (19)-(20).

True value					CSS method					Whittle's method				
d	a_0	a_1	l	ζ	$\hat{d}$	$\hat{a}_0$	$\hat{a}_1$	$\hat{l}$	$\hat{\zeta}$	$\hat{d}$	$\hat{a}_0$	$\hat{a}_1$	$\hat{l}$	$\hat{\zeta}$
								$T = 500$						
0.25	0.6	0.4	3	0.5	0.2486	0.6173	0.4108	3.1308	0.5004	0.2672	0.6073	0.4211	3.1862	0.4759
					(0.0129)	(0.1265)	(0.1258)	(0.3713)	(0.0371)	(0.0436)	(0.1227)	(0.1159)	(0.4010)	(0.0405)
					[0.0169]	[0.1635]	[0.1635]	[0.4991]	[0.0224]	[0.0595]	[0.1576]	[0.1519]	[0.5424]	[0.0256]
0.3	0.2	0.3	3	0.5	0.2983	0.2216	0.3194	3.0741	0.4966	0.3651	0.1949	0.3797	3.2338	0.4397
					(0.0133)	(0.0514)	(0.1186)	(0.3702)	(0.0368)	(0.0723)	(0.0447)	(0.1129)	(0.4383)	(0.0464)
					[0.0171]	[0.0866]	[0.1586]	[0.5109]	[0.0229]	[0.0595]	[0.1576]	[0.1519]	[0.5424]	[0.0253]
0.35	0.8	0.5	3	0.5	0.3479	0.7961	0.4854	3.1979	0.5063	0.3578	0.7956	0.5145	3.1483	0.4041
					(0.0139)	(0.1287)	(0.1241)	(0.3882)	(0.0393)	(0.0444)	(0.1289)	(0.1320)	(0.3336)	(0.0419)
					[0.0178]	[0.1482]	[0.1575]	[1.2128]	[0.0240]	[0.0611]	[0.1496]	[0.1669]	[0.4828]	[0.0268]
								$T = 1000$						
0.25	0.6	0.4	3	0.5	0.2489	0.6283	0.4168	3.0310	0.5028	0.2526	0.6097	0.4194	3.0821	0.4719
					(0.0094)	(0.0943)	(0.0875)	(0.2492)	(0.0278)	(0.0373)	(0.0914)	(0.0936)	(0.2552)	(0.0276)
					[0.0119]	[0.1292]	[0.1179]	[0.3264]	[0.0178]	[0.0523]	[0.1200]	[0.1205]	[0.3306]	[0.0183]
0.3	0.2	0.3	3	0.5	0.2986	0.2081	0.3096	3.0429	0.5021	0.2526	0.6097	0.4194	3.0821	0.4719
					(0.0088)	(0.0311)	(0.0768)	(0.2608)	(0.0271)	(0.0376)	(0.0307)	(0.0795)	(0.2608)	(0.0290)
					[0.0113]	[0.0424]	[0.0991]	[0.3431]	[0.0171]	[0.0497]	[0.0422]	[0.1063]	[0.3412]	[0.0182]
0.35	0.8	0.5	3	0.5	0.3499	0.8077	0.5146	3.0594	0.5002	0.3558	0.7935	0.5141	3.0997	0.5689
					(0.0091)	(0.1039)	(0.0969)	(0.2376)	(0.0278)	(0.0375)	(0.1051)	(0.0969)	(0.2536)	(0.0299)
					[0.0111]	[0.1238]	[0.1233]	[0.2999]	[0.0175]	[0.0599]	[0.1248]	[0.1233]	[0.3303]	[0.0209]

– In these tables, the true parameter values used in the data-generating process are given in the first m columns (m is the number of parameters to be estimated). The estimations of these parameters are given in the next m columns, the mean absolute error (MAE) is given in the row below and the root mean square error (RMSE) is given under the row of MAE.

– In Table I, the results from the k-factor GIGARCH model with conditional normal errors are presented. From this table, we see that all methods perform very well as the MAE and RMSE are in most cases small. In general, the estimates parameters from the CSS approach are better than those given by Whittle method. Indeed the former approach takes into account all the properties of the model through the conditional distribution function.

– **Tables II-IV summarize the simulation results when the conditional distribution is non normal. Here, we present the Student-t with $l = 5$ degrees of freedom, the GED with exponent (or shape parameter) equal to $l = 1.5$ and the skew Student-t distribution with shape parameter equal to $l = 3$ and skew parameter equal to $\zeta = 0.5$. The values of the true and the estimated parameters are also given in these tables. The CSS procedure estimates all parameter simultaneously while the estimates from the Whittle method were obtained from a two-step approach. Notice that for Whittle approach, the distributional parameters are obtained by applying maximum likelihood method to the standardized residuals of the ARCH model.** Results reveal that estimates parameters are satisfactory in the sense that the MSE and the RMSE are very small. We can also observe that the results from the CSS procedure seem to perform better than those obtained by Whittle approach.

– From the results, we observe that, in general, the estimators seem to be unaffected by the presence of ARCH errors. This phenomenon is frequently noted in the literature, Sena *et al.* (2006). The Monte Carlo experiments show the impact of the sample size T on these estimation methods. Indeed when the sample of observations increases significantly (T=1000), the results improve significantly.

5. SUMMARY

In this paper, we have dealt with a special class of long memory models with heteroskedastic noise. Two parameter estimation techniques for k-factor GIGARCH process have been considered. Finite sample behaviors of these methods were studied through Monte-Carlo simulations. It is found that they are relatively comparable in terms of finite sample performance. However, the conditional sum of squares (CSS) approach seems to be more efficient than the Whittle approach even if the conditional distribution for the innovations is Gaussian. The results carried out the fact that the estimator of the long memory is unaffected when there is the presence of ARCH components.

This article focuses on the estimation of generalized long memory time series with conditional heteroskedastic disturbances. Regarding the estimation results when the innovations are skew Student-t (asymmetric), it appears to be interesting to develop and study this new theoritical model in a compagnon paper.

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